






Advances in Transdisciplinary Engineering series

Volume 79

Advances in Machinery, Materials Science and Engineering Application XI

Proceedings of the 11th International Conference (MMSE 2025), Paris, France, 25–27 July 2025

EDITED BY
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ADVANCES IN MACHINERY, MATERIALS SCIENCE AND ENGINEERING APPLICATION XI

Advances in Transdisciplinary Engineering

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Volume 79

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Foreword

The 11th International Conference on Advances in Machinery, Material Science and Engineering Application (MMSE 2025) is the premier international conference in the fields of mechanical engineering, material science and engineering application. This volume includes all papers accepted for presentation at the MMSE 2025 Conference, which was held in Paris, France, from 25 to 27 July. MMSE 2025 is organized by ISAE-SUPMECA, France and Wuhan University, China, co-sponsored by Wuhan University of Science and Technology, China; Huazhong University of Sciences and Technology, China; Wuhan University of Technology, China; China University of Geosciences (Wuhan), China; Wuhan Textile University, China; National University of Singapore, Singapore; Portland State University, USA; Washington University-St. Louis, USA; University of Reims Champagne-Ardenne, France; George Mason University, USA; Laboratoire Quartz, France, and the Institute of Materials, Minerals and Mining (IOM3), UK, among others.

The conference aims to bring together faculty members, leading scientists, academicians, research and graduate scholars, industry professionals, and decision-makers to discuss the latest developments, applications, advanced technologies, and processes in mechanical engineering and advanced materials, with particular focus on the interdisciplinary applications.

The two-day conference in Paris consisted of keynote speeches, scientific presentations, poster presentations and technical discussions. The proceedings of the conference contains 117 high-quality papers selected from 292 submissions, including international contributions from Asia and Europe, representing an acceptance rate of approximately 40%.

The accepted papers highlight the latest developments and research trends from a wide range of disciplines within the scope of the conference, and cover a broad range of topics, including mechanical design; advanced manufacturing technology; applied mechanics; fatigue and creep of materials; corrosion; coatings; electrical power; electronic techniques; energy storage; automation and control system design; robots; shock and vibration; simulation and modeling; machine vision; object detection; failure analysis; chemical engineering; marine engineering; structural engineering; electro-optical technology; autonomous driving technology; and emerging industrial applications and interdisciplinary technology. All contributions were subjected to a rigorous peer review process to ensure academic rigor innovation, and a contribution to the advancement of knowledge.

We would like to express our sincere gratitude to the conference chairs: Prof. Emin Bayraktar, ISAE-SUPMECA/Paris, France; Prof. Seeram Ramakrishna, National University of Singapore, Singapore and Prof. Ephraim Suhir, Life Fellow of IEEE, ASME, SPIE, IMAPS, Fellow of APS, IoP (UK) and SPE and Associate Fellow of AIAA, Portland State University, USA, for their dedication in making this MMSE 2025 a success.

We would also like to express our sincere gratitude to our keynote speakers: Prof. Yaohua Zhu, Hong Kong Polytechnic University, China, Prof. Yunfeng Liu, Zhejiang University of Technology, China, Prof. Weiguo Li, College of Aerospace Engineering,

Chongqing University, China, Prof. Raul Duarte Salgueiral Gomes Campilho, ISEP – School of Engineering, Portugal, Prof. Michael Todinov, Oxford Brookes University, UK, who joined us to present and share their latest findings. Thanks are also due to the MMSE reviewers, authors, and others who contributed to the success of the conference. We appreciate the support and assistance provided by all committee members throughout the event. MMSE 2025 is also indebted to IOS Press for their assistance and support in the publication of this volume.

Finally, on behalf of this MMSE 2025 Committee, and indeed the whole MMSE team, we would like to express our sincere appreciation to all authors and participants for their contributions. We believe that this MMSE 2025 proceedings will serve as an important archival reference for researchers and practitioners in the field. The next 12th International Conference on Advances in Machinery, Material Science and Engineering Application (MMSE 2026) will be held in Wuhan, China, hosted by Huazhong University of Sciences and Technology, China from 26 to 27 July 2026, and we look forward to seeing you in Wuhan next year.

Emin Bayraktar
ISAE-SUPMECA/Paris
27 July 2025

About the Conference

Conference Name
2025 11th International Conference on Advances in Machinery, Materials Science and Engineering Application (MMSE2025)
Conference Location: Paris, France
Date: 25–27 July 2025

Peer Review Statement
Number of submissions: 292
Number of accepted papers: 117
Acceptance rate: 40%

Conference Organization

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Section 1

Mechanical Engineering and Manufacturing Technology

Advances in Machinery, Materials Science and Engineering Application XI
J. Ma et al. (Eds.)

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Vibration Control Using a Quasi-Zero Stiffness Inerter: The Tuning Approach

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Abstract. The use of dynamic vibration absorbers remains one of the most effective methods for mitigating unwanted oscillations in various engineering applications. Nonlinear absorbers, such as those based on Quasi-Zero Stiffness (QZS) and Nonlinear Energy Sinks (NES), offer significant advantages in terms of energy dissipation over a wide frequency range. This paper focuses on a hybrid system composed of a single-degree-of-freedom QZS main structure with an attached NES-type inerter absorber. An analytical approach is presented to derive the frequency-amplitude relation (FAR), accounting for six key dimensionless parameters that describe the system dynamics. Although the general expression for FAR is complex, for a given mechanical configuration of the main structure, it becomes feasible to determine the optimal damping coefficient and nonlinear stiffness of the absorber using a straightforward analytical-numerical method. The practical absence of linear stiffness components enables the absorber to operate efficiently under external harmonic excitation. Numerical integration of the system's equations of motion demonstrates strong agreement with theoretical predictions and confirms the effectiveness of the proposed tuning methodology in suppressing vibrations.

Keywords. Vibration suppression, quasi-zero stiffness, nonlinear energy sink, averaging method, tuning methodology

1. Introduction

The tuned mass damper (TMD) technology is one of the most promising methods to suppress and control unwanted vibrations. It is mechanically simple, cost-effective, has reliable operation and has attracted many efforts of researchers during the last decades [1, 2].

One of the notable areas of research in recent years is the use of the quasi-zero stiffness absorbers (QZS). They are realized by combining the positive stiffness provided by the nonlinear springs and the negative stiffness generated by different sources such as oblique springs [3], oblique inerters [4], the centrifugal forces [5], cam–roller–springs [6], limb-inspired bionic structure which generates negative stiffness via torsion springs [7] etc. In this regard, we also draw attention to the works [8 – 13].

Another perspective is granted by Nonlinear energy sink (NES) which presents an expediently designed nonlinear oscillator without positive linear stiffness. NES, as an essentially nonlinear system, can suppress vibrations over a wide frequency range and is

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widely used in various problems of nonlinear dynamics. The detailed review of this research area is presented in [2]. Let's note some results from recent years. In paper [14] the vibration responses of the system consisting of the linear oscillator coupled with traditional NES and a piecewise spring introduced between the NES and the ground were analyzed theoretically, and the results were verified experimentally. Yang et al. [4] investigated a nonlinear inertance mechanism (NIM) for vibration mitigation. In [15] the Hybrid Vibration Isolator with NES that combines QZS and nonlinear ineritance mechanisms (NIM) for broadband vibration control was studied.

In the present paper we consider the approximate dynamical model of a single DoF QZS main system with an attached NES inerter. The analytical scheme of obtaining frequency-amplitude relation (FAR) is proposed. Due to the presence of six dimensionless parameters this relation is cumbersome, but for given mechanical configuration of the main structure it is possible to find optimal damping and nonlinear stiffness of the inerter to suppress responses in a wide frequency range by a rather simple analytically-numerical way.

2. Formulation of the Problem

We will consider a mechanical model consisting of a single-degree-of-freedom primary mass with the quasi-zero stiffness and attached inertial nonlinear energy sink (NES) [8, 9, 16]. Schematically this system is shown in figure 1.

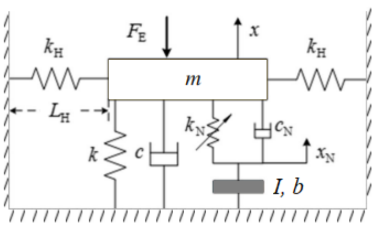

Figure 1. A single DoF model with NES inerter attached.

In order to demonstrate our tuning methodology, we accept some simplifications as it was done in [16], thus our initial dynamic equations read

$$m\ddot{x} + c\dot{x} + kx + 2\left(1 - \frac{L_0}{L_H}\right) k_H x + \frac{L_0}{L_H^3} k_H x^3 + c_N(\dot{x} - \dot{x}_N) + k_N(x - x_N)^3 + F_e\, cos(\omega t) = 0,$$
$$b\ddot{x}_N + c_N(\dot{x}_N - \dot{x}) + k_N(x_N - x)^3 = 0. \quad (1)$$

The dimensionless form of the approximate Eq. (1) is

$$\ddot{x}_1 + 2\zeta\,\dot{x}_1 + \alpha x_1 + \tilde{\beta} x_1^3 + 2\zeta_N(\dot{x}_1 - \dot{x}_2) + K_N(x_1 - x_2)^3 + f_e \cos(\tilde{\Omega}\tau) = 0,$$
$$\mu\ddot{x}_2 + 2\zeta_N(\dot{x}_2 - \dot{x}_1) + K_N(x_2 - x_1)^3 = 0, \quad (2)$$

where

$$x_1 = \frac{x}{L_Q}, x_2 = \frac{x_N}{L_Q}, L_Q = \sqrt{L_0^2 - L_H^2}, \omega_0^2 = \frac{k}{m}, \tau = \omega_0 t, \tilde{\Omega} = \frac{\omega}{\omega_0},$$
$$\zeta = \frac{c}{m\omega_0}, \alpha = 1 + 2\left(1 - \frac{1}{\varepsilon}\right)\kappa_H, \tilde{\beta} = \left[\left(\frac{1}{\varepsilon}\right)^3 - \frac{1}{\varepsilon}\right]\kappa_H, \varepsilon = \frac{L_H}{L_0},$$

$$\kappa_H = \frac{k_H}{k}, \zeta_N = \frac{c_N}{m\omega_0}, K_N = \kappa_N L_Q^2, \kappa_N = \frac{k_N}{k}, \mu = \frac{b}{m}, f_e = \frac{F_e}{L_Q k}. \quad (3)$$

For the convenience of subsequent transformations, we present equations (2) in matrix form

$$2\tilde{\Omega}\boldsymbol{M}\boldsymbol{x}'' + \boldsymbol{D}\boldsymbol{x}' + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{\Phi}(\tau, x_1, x_2).$$

Here

$$\boldsymbol{M} = \begin{pmatrix} \mu+1 & -\mu \\ -\mu & \mu \end{pmatrix}, \boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \boldsymbol{D} = diag(\zeta, \zeta_N), \boldsymbol{K} = diag(\alpha, 0),$$

$$\boldsymbol{\Phi} = \begin{pmatrix} f_e \cos\tilde{\Omega}\tau - \tilde{\beta}x_1^3 \\ \kappa_N x_2^3 \end{pmatrix},$$

where the prime means the derivative on time τ.

With the transformation $x_j = u_j \cos\tilde{\Omega}\tau + v_j \sin\tilde{\Omega}\tau, x_j' = -u_j\tilde{\Omega}\sin\tilde{\Omega}\tau + v_j\tilde{\Omega}\cos\tilde{\Omega}\tau$ $(j = 1, 2)$ and applying the method of averaging (as it was done in [17]), we get the simplified version of the system

$$\tilde{\Omega}\boldsymbol{M}\,\boldsymbol{u}' + \frac{1}{2}\,[\boldsymbol{D}\boldsymbol{u} + (\boldsymbol{M} - \boldsymbol{K})\boldsymbol{v}] = \frac{3}{8}\begin{pmatrix} \tilde{\beta}\,v_1\,(u_1^2 + v_1^2) \\ \kappa_N\,v_2\,(u_2^2 + v_2^2) \end{pmatrix},$$

$$\tilde{\Omega}\boldsymbol{M}\,\boldsymbol{v}' + \frac{1}{2}[(\boldsymbol{K} - \boldsymbol{M})\boldsymbol{u} + \boldsymbol{D}\,\boldsymbol{v}] = \frac{3}{8}\begin{pmatrix} \tilde{\beta}\,u_1\,(u_1^2 + v_1^2) + \frac{4}{3}f_e \\ \kappa_N\,u_2\,(u_2^2 + v_2^2)2 \end{pmatrix}, \quad (4)$$

which do not contain the time dependent terms, as all of them have zero average value over $2\pi/\tilde{\Omega}$ time period.

3. Frequency-Amplitude Relations and Tuning Methodology

The stationary points of system (1) correspond to quasi-periodic solutions of equations (2) and are determined by equalities

$$\zeta\tilde{\Omega}u_1 + \left(\mu\tilde{\Omega}^2 - \alpha + \tilde{\Omega}^2\right)v_1 - \mu\tilde{\Omega}^2 v_2 - \frac{3}{4}\tilde{\beta}v_1(u_1^2 + v_1^2)$$
$$= 0, -\mu\tilde{\Omega}^2 v_1 + \zeta_N\tilde{\Omega}u_2 + \mu\tilde{\Omega}^2 v_2 - \frac{3}{4}\kappa_N v_2(u_2^2 + v_2^2) = 0,$$
$$\left(\alpha - \mu\tilde{\Omega}^2 - \tilde{\Omega}^2\right)u_1 + \zeta\tilde{\Omega}v_1 + \mu\tilde{\Omega}^2 u_2 + \frac{3}{4}\tilde{\beta}u_1(u_1^2 + v_1^2) + f_e = 0, \quad (5)$$
$$\mu\tilde{\Omega}^2(u_1 - u_2) + \zeta_N\tilde{\Omega}v_2 + \frac{3}{4}\kappa_N u_2(u_2^2 + v_2^2) = 0.$$

To simplify the calculations, we temporarily introduce complex variables $z_j = u_j + i\,v_j, \bar{z}_j = u_j - i\,v_j$ and obtain the following system of nonlinear algebraic equations,

$$\left(\alpha - \mu\tilde{\Omega}^2 - \tilde{\Omega}^2\right)z_1 + \mu\tilde{\Omega}^2 z_2 + \frac{3}{4}\tilde{\beta}z_1\rho_1 - i\,\zeta\tilde{\Omega}z_1 = 0,$$
$$\mu\tilde{\Omega}^2 z_1 - \mu\tilde{\Omega}^2 z_2 + \frac{3}{4}\kappa_N z_2\rho_2 - i\,\zeta_N\tilde{\Omega}z_2 = 0, CC = 0, \quad (6)$$

where $\rho_j = z_j\,\bar{z}_j$ $(j = 1, 2)$ and CC denotes complex conjugate counterparts.

System (6) cannot be solve directly with respect to variables $z_1, \bar{z}_1, z_2, \bar{z}_2,$ and in order to obtain the frequency-amplitude relations we use the following "trick". Solving formally equations (6) with respect to these variables (ignoring presence of ρ_1, ρ_2 in them) and write down identities $\rho_j - z_j\,\overline{z_j} = 0, (j = 1, 2)$. After simplification we have two polynomials in ρ_1, ρ_2, namely

$$P_1 = k_{13}\rho_1^3 + k_{12}\rho_1^2 + k_{11}\rho_1 + k_{10},\ P_2 = k_{22}\rho_1^2 + k_{21}\rho_1 + k_{20}, \tag{7}$$

where

$$k_{13} = \beta^2(\kappa^2\rho_2^2 - 2\kappa\Omega\mu\rho_2 + \Omega^2\mu^2 + \zeta_N^2\Omega),$$
$$k_{12} = 2\beta[\kappa^2 (-\Omega\mu + \alpha - \Omega)\rho_2^2 + \kappa\Omega\mu(\Omega\mu - 2\alpha + 2\Omega)\rho_2 + \Omega(-\Omega^2\mu^2 + (\alpha\mu^2 - \zeta_N^2(\mu + 1)\Omega + \alpha\zeta_N^2)],$$
$$k_{11} = \kappa^2\big(\Omega^2\mu^2 + \big[2(\Omega - \mu)\big)\Omega\mu + \zeta^2\Omega + (\Omega - \alpha)^2\big]\rho_2^2 + 2\kappa[-(\Omega - \mu)\Omega^2\mu^2 - ((\Omega - \mu)^2 + \zeta^2\Omega)\Omega\mu]\rho_2 + (\Omega(\zeta + \zeta_N)^2 + (\Omega - \mu)^2)\Omega^2\mu^2 + 2(\Omega - \mu)\zeta_N^2\Omega^2\mu + \zeta^2\zeta_N^2\Omega^2 + \Omega\zeta_N^2(\Omega - \mu)^2,\ k_{10} = [-\kappa^2\rho_2^2 + 2\kappa^2\Omega\mu\rho_2 - (\Omega^2\mu^2 + \zeta_N^2)\Omega^2]f_e^2,$$
$$k_{22} = \beta^2[\kappa^2\rho_2^3 - 2\kappa\Omega\mu\rho_2^2 + (\Omega^2\mu^2 + \zeta_N^2\Omega)\rho_2],$$
$$k_{21} = 2\beta\big[\kappa^2\big(\alpha - \Omega(\mu + 1)\big)\rho_2^3 + \kappa\Omega\mu[(\mu + 2)\Omega - 2\alpha]\rho_2^2 + \big(-\Omega^2\mu^2 + \big(\alpha\mu^2 - \zeta_N^2(\mu + 1)\big)\Omega^2 + \alpha\zeta_N^2\big)\Omega\rho_2\big],$$
$$k_{20} = \kappa^2[(\mu + 1)^2\Omega^2 + (\zeta^2 - 2\alpha\mu - 2\alpha)\Omega + \alpha^2]\rho_2^3 - 2\kappa\mu\Omega\big((\mu + 1)\Omega^2 - (\alpha\mu - \zeta^2 + 2\alpha)\Omega + \alpha^2\big)\rho_2^2 + \Omega(\Omega^3\mu^2 + ((\mu + 1)^2\zeta_N^2 + \zeta\mu^2(2\zeta_N + \mu) - 2\alpha\mu^2)\Omega^2 + [(\zeta^2 - 2\alpha\mu - 2\alpha)\zeta_N^2 + \alpha^2\mu^2)\Omega + \alpha^2\zeta_N^2]\rho_2 - f_e^2\Omega^2\mu^2,$$
$$\Omega = \tilde{\Omega}^2, \beta = \frac{3}{4}\tilde{\beta}, \kappa = \frac{3}{4}\kappa_N.$$

Now we can obtain FAR for ρ_1 as resultant of polynomials P_1, P_2 with respect to variable ρ_2 (or FAR for ρ_2 in a similar way). This polynomial $P(\rho_1)$ has ninth oder on ρ_1 and is bulky enough to be presented here. This polynomial may be considered as implicit function ρ_1 on parameters $\Omega, \zeta_N, \kappa, \varepsilon, \kappa_H, \alpha, \beta, f_e, \mu, \zeta$. Anyway, for given parameter set of primary mass this can provide enough information in order to tune the inerter I. If we wish suppress responses of primary mass in a wide frequency band, we need to minimize the maximum value of ρ_1 by appropriate choice of parameters ζ_N, κ. Geometrically, we have hyper-surface in R^4 $(\rho_1, \Omega, \zeta_N, \kappa)$ and are interested in values of inerter parameters where this h-surface has a "concavity" in ρ_1 direction (figure 2).

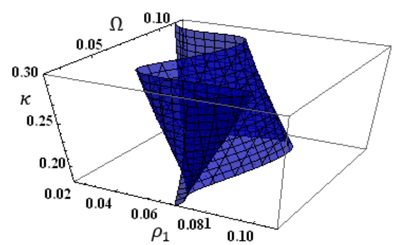

Figure 2. Shape of hyper-surface $P(\rho_1, \Omega, \zeta_N, \kappa) = 0$ for fixed value of $\zeta_N = 0.12$.

Technically the search of optimal pair (ζ_N, κ) may be realized in a following way. Let ρ_* is the maximal height of FA curve (for chosen pair (ζ_{N_0}, κ_0), then straight line $\rho_1 = \rho_* + \varepsilon$ does not intersect this curve, that is system $P(\rho_1, \Omega, \zeta_{N_0}, \kappa_0) = 0$, $\rho_1 = \rho_* + \varepsilon$ is inconsistent. Based on this fact we can use a simple iterative procedure to find optimal pair (ζ_N, κ). For greater clarity we choose parameters of main mass according to [16], which leads to $\varepsilon = 0.7, \kappa_H = 1.167, \alpha = -2.8 \cdot 10^{-4}$, $\beta = 1.3, f_e = 0.015, \mu = 1, \zeta = 0.0189$. Taking it sequentially $\rho_1 = \{0.09, 0.08, 0.072\}$ we seeking where the "hole" in surface $P = 0$ is vanishing (in other words the ray $\zeta_N = \zeta_{N_{opt}}, \kappa = \kappa_{opt}$ does not "touch" this surface). Such a pair $\zeta_N \approx 0.135, \kappa \approx 0.23$ exists for $\rho_1 = 0.072$ and does not exist for $\rho_1 = 0.071$. Thus minimal possible value for responses peak (in terms of ρ_1) is ≈ 0.072. This is illustrated in figure 3.

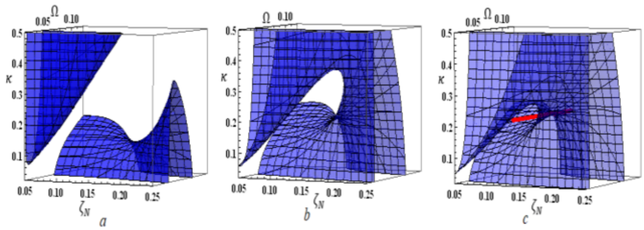

Figure 3. Visualization of search technique for appropriate set on ζ_N, κ. (a): $\rho_1 = 0.09$, (b): $\rho_1 = 0.08$, (c): $\rho_1 = 0.072$.

The corresponding FA curve is shown in figure 4.

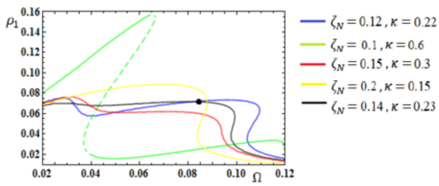

Figure 4. FA curves for different values on inerter damping and nonlinear stiffness.

4. Discussion and Numerical Validation

Let us compare the results obtained on the basis of the proposed scheme with the results of numerical integration of equations (4). As can be seen in figure 3, the maximum response value is achieved at $\Omega \approx 0.82$. Substituting $\widetilde{\Omega} = 0.284$ into equations (5), we find $u_1 = -0.0182, v_1 = -0.2671, u_2 = -0.1773, v_2 = -0.2399$. For frequency values $\widetilde{\Omega} = \{0.2, 0.284, 0.32\}$, as shown in figure 5, the response values (the radius vector of the attraction point) will be somewhat smaller.

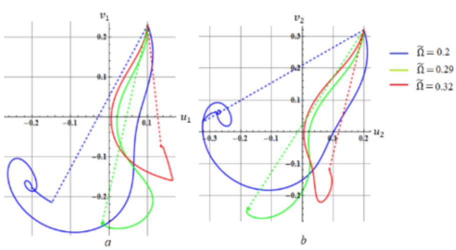

Figure 5. Projections of phase trajectories in variables u, v. Points of attraction correspond to solutions of the system (3.1).

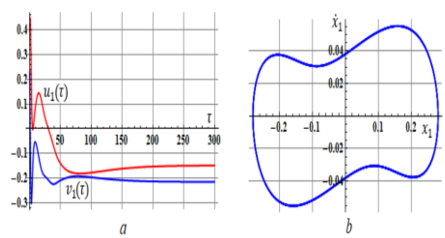

Figure 6. (a) Time histories for $u_1(\tau), v_1(\tau)$; (b) The limit cycle in dimensionless displacement and velocity.

As can be seen from figure 6a, after $\tau \approx 15$ the variables u, v change slowly, which confirms the appropriateness of the averaging procedure. The limit cycle with respect to variables $x_1, \dot{x}_1$ is shown in figure 6b. Note also that for the "empirically" selected pair $\zeta_N = 0.1, \kappa = 0.6$ (the green curve in figure 4), and $\widetilde{\Omega} = 0.25$ the system has three real solutions, two of which correspond to stable stationary points. Accordingly, a pitchfork bifurcation takes place in the frequency range $(0.61, 0.81)$.

5. Conclusion

The article discusses the problem of passive suppression of oscillations caused by external harmonic excitation. The practical absence of a linear component of stiffness is essential, which is achieved by using the QZS of the main structure and NES of the absorber. Analytical scheme to obtain frequency-amplitude relation is proposed. Although in general form (with unknown mechanical parameters) the resulting expression is cumbersome, for a given main structure the search of optimal absorber configuration is simple enough. The results of numerical integration of the equations of motion show good agreement with the calculations. The future work will involve the implementation of an asymptotic approach in order to simplify key relations between mechanical parameters of the system and response magnitudes.

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